Particle Pair Production in Cosmological General Relativity

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Abstract The Cosmological General Relativity (CGR) of Carmeli, a 5-dimensional (5-D) theory of time, space and velocity, predicts the existence of an acceleration $a_0 = c/\tau$ due to the expansion of the universe, where c is the speed of light in vacuum, $\tau = 1/h$ is the Hubble-Carmeli time constant, where h is the Hubble constant at zero distance and no gravity. The Carmeli force on a particle of mass m is $F_c = ma_0$, a fifth force in nature. In CGR, the effective mass density $\rho_{eff} = \rho - \rho_c$, where ρ is the matter density and ρ_c is the critical mass density which we identify with the vacuum mass density $\rho_{vac} = -\rho_c$.

The fields resulting from the weak field solution of the Einstein field equations in 5-D CGR and the Carmeli force are used to hypothesize the production of a pair of particles. The mass of each particle is found to be $m = \tau c^3/4G$, where G is Newton's constant. The vacuum mass density derived from the physics is $\rho_{vac} = -\rho_c = -3/8\pi G\tau^2$. We make a connection between the cosmological constant of the Friedmann-Robertson-Walker model and the vacuum mass density of CGR by the relation $\Lambda = -8\pi G\rho_{vac} = 3/\tau^2$. Each black hole particle defines its own volume of space enclosed by the event horizon, forming a sub-universe.

The cosmic microwave background (CMB) black body radiation at the temperature $T_o = 2.72548\,\mathrm{K}$ which fills that volume is found to have a relationship to the ionization energy of the Hydrogen atom. Define the radiation energy $\epsilon_\gamma = (1-g)mc^2/N_\gamma$, where $(1-g)$ is the fraction of the initial energy mc^2 which converts to photons, g is a function of the baryon density parameter Ω_b and N_γ is the total number of photons in the CMB radiation field. We make the connection with the ionization energy of the first quantum level of the Hydrogen atom by the hypothesis

$$\epsilon_\gamma = \frac{(1-g)\,mc^2}{N_\gamma} = \frac{\alpha^2\mu c^2}{2}, \qquad (1)$$

where α is the fine-structure constant and $\mu = m_p f/(1+f)$, where $f = m_e/m_p$ with m_e the electron mass and m_p the proton mass. We give a model for $g \approx \Omega_b(1+f)m_p/m_n$, where m_n is the neutron mass. Then ratio η of the number of

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baryons N_b to photons N_γ is given by

$$\eta = \frac{N_b}{N_\gamma} \approx \frac{\alpha^2 \Omega_b f m_p / m_n}{2(1+f)[1 - \Omega_b (1+f) m_p / m_n]} \tag{2}$$

with a value of $\eta \approx 6.708 \times 10^{-10}$.

The Bekenstein-Hawking black hole entropy S is given by $S = (kc^3 A)/(4\hbar G)$, where k is Boltzmann's constant, $\hbar$ is Planck's constant over 2π and A is the area of the event horizon. For our black hole sub-universe of mass m the entropy is given by

$$S = \frac{\pi k \tau^2 c^5}{\hbar G}, \tag{3}$$

which can be put into the form relating to the vacuum mass density

$$\rho_{vac} = \frac{\rho_P}{(S/k)}, \tag{4}$$

where the cosmological Planck mass density $\rho_P = -\mathcal{M}_P / L_P^3$. The cosmological Planck mass $\mathcal{M}_P = \sqrt{\sqrt{3/8} \, \hbar c / G}$ and length $L_P = \hbar / \mathcal{M}_P c$. The value of $(S/k) \approx 1.980 \times 10^{122}$.

1 Introduction

Carmeli's Cosmological General Relativity(CGR), [1] and [2,3], is a 5-dimensional (5-D) theory of time, space and velocity. Carmeli predicted [4] an acceleration due to the expansion of the universe

$$a_0 = \frac{c}{\tau}, \tag{5}$$

where c is the speed of light in vacuum and $\tau = 1/h$ is the Hubble-Carmeli time constant where h is the Hubble constant at zero distance and no gravity. The Carmeli force on a particle of mass m is $F_c = m a_0$, a fifth force in nature. As of this writing the published values [5] for $\tau = 4.28 \times 10^{17}$s $= 13.58$ Gyr and $h = 1/\tau = 72.17 \pm 0.84$ km s^{-1} Mpc^{-1}. In this paper, unless noted otherwise, all calculations have used this published value for τ.

We use the weak field solution of CGR to derive a_0. The fields and the Carmeli force are used to hypothesize the production of a particle and its antiparticle. The vacuum mass density ρ_{vac} is defined. By applying the physical laws, the masses and separation distance are determined for the particles. Consequently, a value for $\rho_c = -\rho_{vac}$ is obtained, which is derived from the physics rather than being assumed as was previously done.

2 Geodesic equations in 5-D

The coordinates in 5-D CGR are t for time, (x^1, x^2, x^3) for space, and v for expansion velocity. The line element for a linearized metric is given by

$$\begin{aligned} ds^2 &= (1 + \phi(r)/c^2)(dx^0)^2 \\ &\quad - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \\ &\quad + (1 + \psi(r)/\tau^2)(dx^4)^2, \end{aligned} \tag{6}$$

where $dx^0 = cdt$, c is the speed of light in vacuo, $dx^4 = \tau dv$ and τ is the Hubble-Carmeli time constant. The spatial line element

$$r = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}. \tag{7}$$

The geodesic equation is expressed

$$\frac{d^2 x^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0, \tag{8}$$

$$\alpha, \beta, \mu = 0, 1, 2, 3, 4, \tag{9}$$

where $\Gamma^\mu_{\alpha\beta}$ are the Christoffel symbols and repeated indices are summed (Einstein summation convention.) This describes five equations of motion. By changing the independent parameter s into an arbitray parameter σ, and then letting σ be the coordinate x^1 in one instance and the coordinate x^4 in another instance, as detailed in [4], and then using the Einstein-Infeld-Hoffmann method in the first approximation, (8) reduces to

$$\frac{d^2 x^k}{dt^2} = -\frac{1}{2} \frac{\partial \phi(r)}{\partial x^k}, \tag{10}$$

$$\frac{d^2 v}{dt^2} = 0, \tag{11}$$

$$\frac{d^2 x^k}{dv^2} = -\frac{1}{2} \frac{\partial \psi(r)}{\partial x^k}, \tag{12}$$

$$\frac{d^2 t}{dv^2} = 0, \tag{13}$$

$$k = 1, 2, 3. \tag{14}$$

The first integral of (11) gives

$$\frac{dv}{dt} = a_0, \tag{15}$$

where a_0 is a constant. The first integral of (13), assuming that $a_0 \neq 0$, gives

$$\frac{dt}{dv} = \frac{1}{dv/dt} = a_0^{-1}. \tag{16}$$

Then (12) can be expressed

$$\frac{d^2x^k}{dv^2} = \left[\frac{d}{dt}\frac{dx^k}{dv}\right]\frac{dt}{dv}, \tag{17}$$

$$= \left(\frac{d^2x^k}{dt^2}\right)\left(\frac{dt}{dv}\right)^2, \tag{18}$$

$$= \left(\frac{1}{a_0^2}\right)\left(\frac{d^2x^k}{dt^2}\right), \tag{19}$$

where we have substituted from (16) for dt/dv in (18) to get (19). Finally, from (10), (12) and (19) we get

$$\frac{\partial\psi(r)}{\partial x^k} = \left(\frac{1}{a_0^2}\right)\frac{\partial\phi(r)}{\partial x^k}. \tag{20}$$

To show that $\psi(r) = (\tau^2/c^2)\phi(r)$, which from (20) implies that $a_0 = \pm c/\tau$, the solutions to the field equations are needed.

3 Einstein field equations in 5-D

The Einstein field equations in 5-D are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}, \tag{21}$$

$$\mu, \nu = 0, 1, 2, 3, 4, \tag{22}$$

where $R_{\mu\nu}$ is the Ricci tensor, R its contraction, $g_{\mu\nu}$ is the metric, $\kappa = 8\pi G/c^2\tau^2$ and the stress-energy tensor is given by

$$T_{\mu\nu} = \rho_{eff}u_\mu u_\nu + p\left(u_\mu u_\nu - g_{\mu\nu}\right), \tag{23}$$

where

$$\rho_{eff} = \rho - \rho_c \tag{24}$$

is the effective mass density, with ρ the matter density and ρ_c the critical mass density. Also, p is the pressure and $u_\mu = u^\mu = (1, 0, 0, 0, 1)$ is the velocity vector. Solving [4, 2, 3] the field equations (21) for the metric defined by (6), the equations for the fields $\phi(r)$ and $\psi(r)$ are found to be

$$\nabla^2\phi(r) = \left(c^2/\tau^2\right)\nabla^2\psi(r) = -16\pi G\rho_{eff}, \tag{25}$$

The solutions to (25) for the non-homogeneous case are given by

$$\phi(r) = \left(c^2/\tau^2\right)\psi(r) = -\rho_{eff}\left(\frac{8\pi G}{3}\right)r^2, \tag{26}$$

and for the homogeneous case by

$$\phi(r) = \left(c^2/\tau^2\right)\psi(r) = -\frac{2Gm}{r}, \tag{27}$$

where m is a constant mass. The general solution is the sum of (26) and (27),

$$\phi(r) = -\rho_{eff}\left(\frac{8\pi G}{3}\right)r^2 - \frac{2Gm}{r}, \tag{28}$$

$$\psi(r) = -\rho_{eff}\left(\frac{8\pi G}{3}\right)\frac{\tau^2 r^2}{c^2} - \frac{2Gm\tau^2}{c^2 r}. \tag{29}$$

4 The Carmeli force

By combining the three spatial coordinates (x^1, x^2, x^2) into radial r in (20) and by (26) and (27),

$$\nabla\psi(r) = \left(\frac{\tau^2}{c^2}\right)\nabla\phi(r). \tag{30}$$

Assuming $\nabla\psi(r) \neq 0$, (15), (20) and (30) imply

$$a_0^2 = c^2/\tau^2, \tag{31}$$

$$\frac{dv}{dt} = a_0 = \pm c/\tau. \tag{32}$$

In summary, the 5-D geometrical system implies the geodesic equations (10)-(13). They indicate that, for very great distances from a source of mass, where the acceleration $\nabla\phi(r) \approx 0$ though not zero, there is still the effect of the constant acceleration a_0 caused by expansion in the velocity dimension which is directed along the radial spatial direction. The Carmeli force on a particle of mass m in vector notation is given by

$$\mathbf{F_c} = ma_0\mathbf{r}/r. \tag{33}$$

Hence, the resulting time rate of change in displacement $\mathbf{r}$ of a particle in 5-D is expressed[4] in vector notation by

$$\frac{d^2\mathbf{r}}{dt^2} = -\nabla\phi(r) + a_0\mathbf{r}/r, \tag{34}$$

where the actual value of a_0, either $+c/\tau$ or $-c/\tau$ is determined by the physics. Note that the Carmeli force occurs only when the gradient of the fields $\phi(r)$ and $\psi(r)$ are non-zero. Since the fields propagate at speed c, so must the Carmeli force field.

5 Particle pair production

In 5-D CGR there are only two parameter constants, a maximum speed c, the speed of light in vacuum, and a maximum time τ, the age of the universe. In the following discourse, we use the term *matter* in a general way to include all particles of non-zero and zero rest mass. Likewise, we use the term *vacuum* to refer to that which is unmaterialized but which has energy.

By the definition of $\rho_{eff} = \rho - \rho_c$ found in [1] and [2], Sect. 4.3.1., we have the difference of two positive quantities ρ and ρ_c. It is common in CGR to associate

ρ_c with ρ_{vac}[6,5]. If the vacuum density equals the critical density, $\rho_{vac} = \rho_c$, as is the usual definition, then the effective mass density would be $\rho_{eff} = \rho - \rho_{vac}$. But what is the physical process that requires one to subtract the densities? Is it not more fundamental to define the effective mass density ρ_{eff} as the sum of all types of mass densities positive and negative? In this way we can look at the vacuum density as a negative mass density which is added to the positive mass density to obtain an effective density. Even though the original intent may not have been just this idea, we are lead to this point of view by the physical concept. We will show subsequently that the vacuum can indeed be described by a negative mass density when we consider entropy. Hence, in this paper, the effective mass density ρ_{eff} is defined by

$$\rho_{eff} = \rho + \rho_{vac}, \tag{35}$$

where $\rho_c = -\rho_{vac}$.

Before any particles and vacuum were produced the universe is assumed to have been empty with zero total energy. Thus, by energy conservation the total energy must always be zero. Suppose that a pair of electrically neutral particles is produced along with the vacuum. By the laws of quantum physics each particle must be the antiparticle of the other one in order to conserve all quantum numbers. This means, among other things, that each particle has the same mass m. For clarity we assume that one of the particles is composed of $(x+y)q$ amount of matter and $x\bar{q}$ amount of antimatter such that $m = x(q + \bar{q}) + yq$ and the antiparticle $\bar{m} = x(q + \bar{q}) + y\bar{q}$. Summing the masses produces $m + \bar{m} = (2x+y)q + (2x+y)\bar{q}$, showing an equal amount of matter and antimatter for the two particles combined and all quantum numbers are conserved.

Assume that the particles are produced with zero relative velocity (no kinetic energy) at a distance r_0 apart, with the vacuum filling the finite universe. We make our observations from a reference frame with origin fixed to the center of one of the particles. Initially, at the time in the past $t = \tau$ when the particles and vacuum are produced, before the gravitational fields of the particles have begun to propagate at the speed c, the only energies are that of the two rest masses and the vacuum, so these must sum to zero by energy conservation.[1] This is the energy from a *global* perspective. The particles occupy the lowest energy state of the system. It is assumed that the particles and the vacuum are produced instantaneously. Hence, the total energy of the two particles and the vacuum is

$$2mc^2 + \rho_{vac}\left(\frac{4}{3}\pi R_S^3\right)c^2 = 0, \tag{36}$$

where R_S is the (Schwarzschild) radius of the universe and we used the flat space Euclidean volume. Given this radius, the two particle matter density is given by

$$\rho = \frac{2m}{(4/3)\pi R_S^3}. \tag{37}$$

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[1] If the initial energy was non-zero, the non-zero amount would have to be in the form of mass or vacuum since these are the only physical forms we know about. Suppose the initial matter mass and vacuum mass were given by $2m + m_{vac} = m_i + m_{vac_i}$. Since the assumed model is only for a two particle and vacuum system, we could bring everything over to the left hand side and have the new initial condition be $2(m - m_i/2) + (m_{vac} - m_{vac_i}) = 0$. Hence, the initial energy being zero is a logical result of the assumed model.

Eqs. (35)-(37) imply that initially

$$\rho_{eff} = \rho + \rho_{vac} = 0. \tag{38}$$

In CGR, (38) is the condition for flat space, which implies that Newton's laws are valid as a first approximation. Also, since $\rho_{eff} = 0$, there is only the homogeneous solution to consider for (25) which is given by (27).

The next calculation is made when the field from the remote particle has arrived at the origin of the local particle reference system. Although the gravitational field and the Carmeli force field will take a long time to propagate at speed c over distance r_0, we do our calculations just when it has arrived at time $t = \tau - r_0/c$. (To make this evaluation after billions of years have elapsed we must assume, just for this calculation, that the particles are absolutely stable.) Prior to the field arrival there is no net force acting on either particle. Analysis is made relative to the line r joining the particles and it is assumed that the initial forces on each particle will be small enough so that we can make the weak field approximation for 5-D CGR.

We analyze the forces upon and energy of the particle in our local system. The forces are due to the Newtonian potential and the Carmeli force. The energies are potential energy, kinetic energy (assumed zero), and rest energy. Define the effective potential energy function for the particle comprised of gravitation and Carmeli terms,

$$\Phi_{eff}(r) = -m^2 G/r - ma_0 r. \tag{39}$$

Taking the negative gradient of this potential, the total force $\mathbf{F}(r)$ on the particle is

$$\mathbf{F}(r) = -\nabla\Phi_{eff}(r) = -\frac{m^2 G}{r^2}\hat{\mathbf{r}} + ma_0\hat{\mathbf{r}}, \tag{40}$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$, $\mathbf{r} = (x^1, x^2, x^3)$. By Newton's third law, since there are no other forces, the forces due to gravity and the Carmeli force cancel at $r = r_0$,

$$\mathbf{F}(r_0) = 0\hat{\mathbf{r}}. \tag{41}$$

The total energy $E(r)$ for the particle at rest is

$$E(r) = \Phi_{eff}(r) + mc^2 = -m^2 G/r - ma_0 r + mc^2, \tag{42}$$

where we have added the rest energy as the integral constant. The energy is zero at location r_0,

$$E(r_0) = 0. \tag{43}$$

With $a_0 = +c/\tau$, (40)-(43) can be solved for m and r_0 yielding

$$m = \frac{\tau c^3}{4G}, \tag{44}$$

$$r_0 = \frac{c\tau}{2}. \tag{45}$$

Considering the volume containing both particles of mass $2m$, the Schwarzschild radius R_S is

$$R_S = \frac{2G(2m)}{c^2} = \frac{2G}{c^2}\left(\frac{\tau c^3}{2G}\right) = c\tau. \tag{46}$$

The volume V_S of the combined masses is

$$V_S = \frac{4\pi}{3} c^3 \tau^3. \tag{47}$$

In section (6) we will determine the volume of the hemisphere of one of the black holes and it will be seen to be half of the volume used in (47). With the radius (46) in (37) we get the (two particle) matter density

$$\rho = \frac{3}{8\pi G \tau^2}, \tag{48}$$

and by (38) the vacuum mass density

$$\rho_{vac} = \frac{-3}{8\pi G \tau^2}. \tag{49}$$

With $\tau = 4.28 \times 10^{17}$s and the standard value for G, the value of $\rho_c = -\rho_{vac} = 3/8\pi G \tau^2 \approx 9.77 \times 10^{-30}$gm/cm^3 is obtained from the physics rather than by assumption. The particle mass $m \approx 4.32 \times 10^{55}$gm $\approx 2.17 \times 10^{22}$ M$_\odot$. Since the particles are each so massive we conclude that each is a black hole [7]. Furthermore, by the laws of particle physics, each must be the antiparticle of the other.

There is no cosmological constant in CGR. However, we can relate the CGR vacuum density to the cosmological constant Λ of standard Friedmann-Robertson-Walker (FRW) cosmology [6] by

$$\Lambda = -8\pi G \rho_{vac} = \frac{3}{\tau^2}, \tag{50}$$

where the minus sign is required because in this paper the vacuum density is negative. From this we get a value of $\Lambda \approx 1.64 \times 10^{-35} s^{-2}$.

6 Energies, particle counts and constants

Let us look at energies and particle numbers. The mean total energy density in all frequencies of the radiation in a black body at a temperature T is well known[8] and given by

$$u(T) = \frac{\pi^2 k^4 T^4}{15 c^3 \hbar^3}. \tag{51}$$

Consider the volume V_m of the sub-universe of one of the black hole particles of mass m given by (44). Since the interacting black hole event horizons are so close together they are deformed as described by Fig. 2 (c) of [7]. Referring to Fig. 1 of [7], $z_1 - z_2 = c\tau/2$, $(\mu_1 + \mu_2)/2 = c\tau/2$, giving $\Delta z = z_1 - z_2 - (\mu_1 + \mu_2)/2 = 0$. For this extreme case the black hole volume is a hemisphere with radius

$$R_m = \frac{(2)\, 2Gm}{c^2} = c\tau. \tag{52}$$

Then the hemispheric volume of the sub-universe is $V_m = (2/3)\pi R_m^3 = (2/3)\pi c^3 \tau^3$. Then the total energy E_{cmb} of the cosmic microwave background (CMB) radiation (51) at the present CMB temperature T_o is given by

$$E_{cmb} = u(T_o)\, V_m = \frac{2\pi^3 \tau^3 k^4 T_o^4}{45 \hbar^3}. \tag{53}$$

For the current CMB temperature $T_o = 2.72548\,\mathrm{K}$ we get a total energy of $E_{cmb} = 1.85 \times 10^{72}\mathrm{erg}$ which is equivalent to $1.03 \times 10^{18}\mathrm{M_\odot}$.

The mean energy of the photons in a black body at temperature T is given by $\zeta\,k\,T$, where $\zeta = 2.70117803$ is the black body mean energy coefficient[9]. The number N_γ of photons in the CMB at the current temperature T_o is then given by

$$N_\gamma = \frac{E_{cmb}}{\zeta\,k\,T_o} \approx 1.82 \times 10^{87}. \tag{54}$$

6.1 Black body at the ionization epoch

An interesting discovery is made by asking what was the average energy ϵ_γ of the CMB photons when they had a total energy which was equal to the initial particle rest energy mc^2? Specifically, that average energy is given by

$$\epsilon_\gamma \approx \frac{mc^2}{N_\gamma} \approx 13.32\,\mathrm{eV}, \tag{55}$$

which is $98\,\%$ of the ionization energy $13.6\,\mathrm{eV}$ of the Hydrogen atom. Since the CMB is thought to be directly related to the ionization of the Hydrogen atom, it is plausible that ϵ_γ, when defined properly, is exactly equal to the ionization energy of the first quantum level of the Hydrogen atom. We make the hypothesis, with justification given in section (6.2), that

$$\epsilon_\gamma = \frac{(1-g)\,mc^2}{N_\gamma} = \frac{\alpha^2 \mu c^2}{2}, \tag{56}$$

where $(1-g)$ is the fraction of the initial energy mc^2 which converts to photons during the big bang, g is a function of the baryon density parameter Ω_b, α is the fine-structure constant and $\mu = m_e m_p/(m_e + m_p)$ is the reduced electron mass in the Hydrogen atom, with m_e and m_p the masses of the electron and proton, respectively. Combining (44), (53), (54) and (56) and simplifying we obtain

$$\epsilon_\gamma = \frac{45\zeta\,(1-g)\,c^5\hbar^3}{8\pi^3 G\tau^2 k^3 T_o^3} = \frac{\alpha^2 \mu c^2}{2}. \tag{57}$$

When the temperature of the universe was at the $13.6\,\mathrm{eV}$ energy level it is expected that by and large there were no more baryon or lepton productions. We therefore model the plasma as containing only protons, electrons, neutrinos and photons in interaction. In this plasma phase the average kinetic energy is small compared to the rest energies of the proton and electron, so we will ignore the neutrino in this model, as a first approximation. Then we can say that

$$N_\gamma\epsilon_\gamma \approx mc^2 - g_b N\,(m_p + m_e)\,c^2, \tag{58}$$

where we assume that $N = m/m_n$ is the number of baryons in the original mass m with m_n the neutron mass, $g_b = N_b/N$ is the fraction of baryons (and leptons) which survived the annihilations during the big bang and N_b is the number of

baryons after the annihilations. Substituting these values for m and N in (58) and simplifying we have

$$N_\gamma \epsilon_\gamma \approx mc^2 \left[1 - g_b \left(\frac{m_p + m_e}{m_n} \right) \right]. \tag{59}$$

The remnant baryonic mass fraction g_b has the relationship to (48), the original matter density ρ, by

$$g_b = \frac{N_b m_n}{m} = \frac{N_b m_n}{\rho V_m} = \left(\frac{N_b m_n}{V_m} \right) \frac{1}{\rho} = \frac{\rho_b}{\rho}, \tag{60}$$

where ρ_b is the baryon matter density at the current epoch, ρ is the original matter density (48) and V_m is the sub-universe volume. Since the original matter density equals the critical density $\rho = \rho_c$, then

$$g_b = \frac{\rho_b}{\rho_c} = \Omega_b, \tag{61}$$

where Ω_b is the baryon matter density parameter at the current epoch. Comparing (59) and (56), and substituting for g_b from (61) we have

$$g \approx \Omega_b \left(\frac{m_p + m_e}{m_n} \right). \tag{62}$$

Substituting values for the parameters in (62), with $\Omega_b = 0.0443$ from [10] we have

$$g = 0.04426. \tag{63}$$

The fact that $g_b = \Omega_b$ is consitent with the findings of [5] where it was reported that the matter density $\Omega_m \approx \Omega_b$.

For the radiation field at the time of the Hydrogen ionization epoch of the big bang, assuming that it was a black body, the average energy would be $\epsilon_\gamma = \zeta k T_\gamma = \alpha^2 \mu c^2 / 2$, from which we get the ionization black body temperature

$$T_\gamma = \frac{\alpha^2 \mu c^2}{2 \zeta k} \approx 5.84 \times 10^4 \text{K}. \tag{64}$$

The Hubble-Carmeli time constant τ is difficult to measure so let us find its value in terms of all the other parameters which, except for g, we know more precisely. Inverting (57) to obtain τ we have

$$\tau = \sqrt{\frac{45 \zeta (1 - g) c^3 \hbar^3}{4 \pi^3 G \mu \alpha^2 k^3 T_o^3}}. \tag{65}$$

Substituting values for the parameters in (65), with $g = 0.04426$ at the current epoch, we obtain $\tau \approx 4.144 \times 10^{17}$s which corresponds to $h = 74.47 \,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$.

Given that, the cosmological constant (50)

$$\Lambda = \frac{3}{\tau^2} = \frac{4 \pi^3 G \mu \alpha^2 k^3 T_o^3}{15 \zeta (1 - g) c^3 \hbar^3}. \tag{66}$$

With this expression and again with $g = 0.04426$, $\Lambda \approx 1.747 \times 10^{-35} s^{-2}$.

The initial volume of the black hole sub-universe is V_m. It is possible that the black holes sub-universes are in a static condition as described in [7]. The fact that we get good results with very little tweaking of the single parameter τ is amazing to say the least. Tentatively, this could suggest that the expansion of the universe is actually the filling of the static volume V_m of the sub-universe with radiation and matter from the Hydrogen ionization era as well as from all subsequent radiations from galaxies, stars, etc. In cosmological special relativity ([2], p. 19), the cosmic time is a relative concept. Radiation of wavelength λ_e emitted at a cosmic time t in the past will be redshifted to wavelength λ_o at the present epoch time 0 by the relation

$$\frac{\lambda_o}{\lambda_e} = \sqrt{\frac{1 + t/\tau}{1 - t/\tau}}. \tag{67}$$

6.2 Ratio of the number of baryons to CMB photons

We want to substantiate the claim made in (56). Using (58), suppose we re-write it in the form

$$\epsilon_\gamma \approx \left(\frac{N_b m_n c^2}{g_b N_\gamma} \right) \left[1 - g_b \left(\frac{m_p + m_e}{m_n} \right) \right] = w \frac{\alpha^2 \mu c^2}{2}, \tag{68}$$

where we substituted for $m = N_b m_n / g_b$, and the factor w allows us to assert that the statement is certainly true for some w. Assume $w = 1$.

Express the electron mass as a fraction f of the proton mass by $m_e = f m_p$. Then the reduced mass of the electron in the Hydrogen atom is

$$\mu = \frac{f m_p}{(1 + f)}. \tag{69}$$

Solving (68) for $\eta = N_b/N_\gamma$ and substituting $\Omega_b = g_b$ from (61) we have

$$\eta = \frac{N_b}{N_\gamma} \approx \frac{\alpha^2 \Omega_b f m_p / m_n}{2 (1 + f) [1 - \Omega_b (1 + f) m_p / m_n]}. \tag{70}$$

Substituting values in (70), with $\Omega_b = 0.0443$ from [10], gives the ratio of baryons to photons $\eta \approx 6.708 \times 10^{-10}$, which is just outside of the range of values ($\eta_{10} = 4.7 - 6.5$) from the big-bang nucleosynthesis review[11].

From the previous section (6.1) the ratio of ionization epoch model prediction to theoretical value was $0.98 = 1 - (13.6 - 13.32)/13.6$. For the baryon to photon number ratio the model prediction to big bang nucleosynthesis predition was $0.97 = 1 - (6.708 - 6.5)/6.5$. The combined confidence is $0.95 = 0.98 \times 0.97$ that the value of $w = 1$, roughly speaking.

6.3 Size of the black body at the ionization epoch

We want to determine the physical volume V_γ of the black body which enclosed the cosmic background radiation (CBR) at the ionization epoch when it was at

the higher temperature T_γ. We assume the following basic model for the transformation of the energy from one temperature epoch to another,

$$\frac{E_i}{V_i} = f(T_i, T_f)\frac{E_f}{V_f},$$
(71)

where E_i, V_i and E_f, V_f are the initial and final energy and volume, respectively and $f(T_i, T_f)$ is a transformation function of the initial and final temperatures T_i and T_f, respectively. For a black body of photons the total energy is given by $E = u(T)V$ where $u(T)$ is the black body energy density (51) at temperature T and V is its volume. Applying this to our model (71) and after manipulation we have

$$f(T_i, T_f) = \frac{T_i^4}{T_f^4}.$$
(72)

This is the transformation law that preserves the black body distribution as the radiation expands[12]. Since we are interested in the volume change, substitute (72) into (71) and manipulate it to get the result

$$V_f = \left(\frac{T_i^4}{T_f^4}\right)\frac{E_f V_i}{E_i} = \left(\frac{T_i^4}{T_f^4}\right)\frac{E_f}{u(T_i)},$$
(73)

where we made the substitution $u(T_i) = E_i/V_i$. Since $E_f = (1-g)mc^2$, by definition of the CBR black body energy at the ionization epoch, with $T_i = T_o$, $T_f = T_\gamma$ given by (64) and $V_\gamma = V_f$ we obtain a volume

$$V_\gamma = \left(\frac{T_o^4}{T_\gamma^4}\right)\frac{(1-g)\,mc^2}{u(T_o)},$$
(74)

Substituting numerical values for the various parameters into (74), with $g = 0.04426$ we obtain for the ionization volume $V_\gamma \approx 4.23 \times 10^{71}\mathrm{cm}^3$, which corresponds to a radius of $R_\gamma \approx 4.61 \times 10^{23}\mathrm{cm} \approx 0.149\,\mathrm{Mpc}$. The cosmological redshift z of this ionization epoch is given by $(1 + z) = T_\gamma/T_o \approx 2.14 \times 10^4$.

7 Entropy of the black hole sub-universe

The entropy S of a Bekenstein-Hawking black hole [13,14] is given by

$$S = \frac{kc^3 A}{4\hbar G},$$
(75)

where k is Boltzmann's constant, and A is the surface area of the event horizon. The surface area of the black hole is given by $A = 4\pi R^2$ where R is the radius of the event horizon. The radius of the event horizon for one black hole sub-universe is given by (52), where $R = R_m = c\tau$. Then, the black hole entropy (75) of the sub-universe is given by

$$S = \frac{kc^3\left(4\pi c^2\tau^2\right)}{4\hbar G} = \frac{\pi k\tau^2 c^5}{\hbar G}.$$
(76)

Manipulating (76) we can get it into the form

$$\frac{1}{(S/k)}\left(\frac{-3c^5}{8\hbar G^2}\right) = \frac{-3}{8\pi G\tau^2} = \rho_{vac}. \tag{77}$$

The second factor on the l.h.s. of (77) can be expressed as the cosmological Planck mass density ρ_P in terms of the cosmological Planck mass $\mathcal{M}_P$ and length $\mathcal{L}_P = \hbar/\mathcal{M}_Pc$ defined by

$$\rho_P = \frac{-\mathcal{M}_P}{\mathcal{L}_P^3} = \frac{-3c^5}{8\hbar G^2}. \tag{78}$$

Note that for clarity we have attached the negative sign outside of the mass symbol but the implication is for a negative mass $\mathcal{M}_P$. Solving (78) for $\mathcal{M}_P$ gives

$$\mathcal{M}_P = \sqrt{\sqrt{\frac{3}{8}}\frac{\hbar c}{G}}. \tag{79}$$

Substituting ρ_P into (77) and rearranging we have for the vacuum mass density

$$\rho_{vac} = \frac{\rho_P}{(S/k)}. \tag{80}$$

Substituting values for the parameters in (76) we obtain

$$(S/k) \approx 1.980 \times 10^{122}. \tag{81}$$

The standard Planck mass is $m_P = \sqrt{\hbar c/G}$, so that the cosmological Planck mass $\mathcal{M}_P \approx 0.7825\,m_P$.

8 Discussion

Since no physical laws are violated during the particle, antiparticle production process which we have modeled, the particles are deemed to be real rather than virtual. Each particle with its event horizon constitutes a sub-universe with no mixing of matter between sub-universes, although gravitational interactions occur between event horizons as we have modeled. In other words, the gravitational field of the other sub-universe should cause effects in our sub-universe.

Observations of the cosmos show that our universe is composed of baryons and leptons with undetectable amounts of antibaryons and antileptons. In the standard big bang theory this indicates a violation of the conservation of baryon and lepton numbers. Standard physics does not have a clear cut reason for this violation [15]. In our model, both particles in separate sub-universes take part in ensuring that the quantum numbers are conserved across universes. The particle in our sub-universe would have been composed of a mixture of more quarks, $(x + y)q$, than antiquarks, $x\bar{q}$, and visa-versa for the black hole particle in the other sub-universe.

9 Conclusion

We assumed that the initial state of the universe was empty, having zero energy. We hypothesized that a particle and its antiparticle were produced along with the vacuum. Similar to the standard model, matter has positive energy, but energy conservation requires in our model that the vacuum have negative energy. In the weak gravity field model, by conservation of energy and Newton's laws we were able to derive the mass of each particle $m = \tau c^3/4G$, the initial separation between the particles $r_0 = c\tau/2$ and the vacuum mass density $\rho_{vac} = -\rho_c = -3/8\pi G\tau^2$. Although there is no cosmological constant in CGR, from the vacuum density we get a corresponding expression for the standard model cosmological constant $\Lambda = 3/\tau^2$.

By analyzing the CMB radiation energy and comparing it to the rest energy of the original black hole mass we found a relation between the radiation and particle rest energy. Since the average radiation energy $\epsilon_\gamma \approx 13.6\,\text{eV}$, which is the ionization energy of the Hydrogen atom, we made the assumption that ϵ_γ is exactly equal to the first quantum level ionization energy of the Hydrogen atom when defined as $\epsilon_\gamma = (1 - g)m\,c^2/N_\gamma = \alpha^2\,\mu\,c^2/2$, with g related to the baryon density parameter Ω_b. We were able to express the parameters τ and Λ in terms of Ω_b, but this does not imply that they are dependent on that parameter. Instead, the dependent parameter is the CMB temperature T_o, for which by rearranging (65) we have

$$T_o = \left[\frac{45\zeta\,(1 - g)\,c^3\hbar^3}{4\pi^3 G\mu\alpha^2\tau^2 k^3} \right]^{1/3}. \tag{82}$$

With the published value of $\tau = 4.28 \times 10^{17}$s and for $g = 0.04426$ we get a predicted value of $T_o = 2.67\,\text{K}$, which is $98\,\%$ of the experimental value.

Finally, by investigating black hole thermodynamics we derived the Bekenstein-Hawking entropy S of the black hole sub-universe and from that we determined the relation between the vacuum mass density and the cosmological Planck vacuum density, $\rho_{vac} = \rho_P/S/k$. Is this relevant to the cosmological constant problem of the standard model?

Acknowledgments

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